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which there is question the "force *acceleratrice*" of the planet m will be constantly directed towards the sun M and equal to the sum of the two forces $\frac{fM}{r^2}$ and $\frac{fm}{r^2}$; if then we wish to express it by $\frac{\mu}{r^2}$ as in number 225 it is necessary to take $\mu = f(M+m)$."

SOLUTION OF A PROBLFM.

BY MARCUS BAKER, U. S. COAST SURVEY.

If the centers of the escribed circles to any plane triangle be joined two and two a new triangle is formed, which call the *first derived triangle*; if in like manner the centers of the escribed circles to the first derived triangle be joined two and two another triangle is formed, which call the *second derived triangle.* Let this process be continued till we reach the *n^{th} derived triangle.* It is repuired to determine an expression for the area of this n^{th} derived triangle in terms of functions of the original triangle.

Let $\triangle_0$ represent the original triangle, K_0 its area, s its semiperimeter and R the radius of its circumscribing circle; also let $\triangle_n$, K_n, s_n and R_n represent corresponding functions of the n^{th} derived triangle.

Let A, B, C represent the angles and a, b, c the sides of the original triangle, and O', O'', O''' the centers of the first set of escribed circles; then is $O''A\sin\frac{1}{2}A = s-c$ and $O'''A\sin\frac{1}{2}A = s-b$ whence $O''O''' = a\div\sin\frac{1}{2}A$;

$$\therefore\quad \left.\begin{aligned} O''O''' = \frac{a}{\sin\frac{1}{2}A} = 4R\cos\tfrac{1}{2}A;\quad O'''O' &= \frac{b}{\sin\frac{1}{2}B} = 4R\cos\tfrac{1}{2}B;\\ O'\,O'' &= \frac{c}{\sin\frac{1}{2}C} = 4R\,\mathrm{sos}\,\tfrac{1}{2}C \end{aligned}\right\}. \quad (1)$$

$$K_1 = \frac{O'O''.O''O'''.O'''O'}{4R_1} = \frac{64R^3\cos\frac{1}{2}A\cos\frac{1}{2}B\cos\frac{1}{2}C}{8R}$$
$$= 8R^2\cos\tfrac{1}{2}A\cos\tfrac{1}{2}B\cos\tfrac{1}{2}C.$$

But $4R\cos\frac{1}{2}A\cos\frac{1}{2}B\cos\frac{1}{2}C = s$ and therefore

$$K_1 = 2Rs. \qquad (2)$$

It is an immediate inference from (2) that $K_2 = 2R_1s_1$, or generally that

$$K_n = 2R_{n-1}\,s_{n-1}\,. \qquad (3)$$

From an examination of the figure we see that the angles of the first derived triangle are the arithmetic means of the angles of the original triangle taken two and two. The second derived taiangle stands in the same relation to the first derived triangle that the first derived triangle stands to the original triangle, i. e., its angles are the arithmetic means of the angles of the next preceding triangle taken two and two.

The angles of the successive triangles will therefore have the following values:—

$\triangle$	A	B	C
$\triangle_1$	$(\frac{1}{2})(\pi - A)$	$(\frac{1}{2})(\pi - B)$	$(\frac{1}{2})(\pi - C)$
$\triangle_2$	$(\frac{1}{2})^2(\pi + A)$	$(\frac{1}{2})^2(\pi + B)$	$(\frac{1}{2})^2(\pi + C)$
$\triangle_3$	$(\frac{1}{2})^3(3\pi - A)$	$(\frac{1}{2})^3(3\pi - B)$	$(\frac{1}{2})^3(3\pi - C)$
$\triangle_4$	$(\frac{1}{2})^4(5\pi + A)$	$(\frac{1}{2})^4(5\pi + B)$	$(\frac{1}{2})^4(5\pi + C)$
$\triangle_5$	$(\frac{1}{2})^5(11\pi - A)$	$(\frac{1}{2})^5(11\pi - B)$	$(\frac{1}{2})^5(11\pi - C)$
$\triangle_6$	$(\frac{1}{2})^6(21\pi + A)$	$(\frac{1}{2})^6(21\pi + B)$	$(\frac{1}{2})^6(21\pi + C)$
. .	. . . . . .	. . . . . .	. . . . . .
$\triangle_n$	$(\frac{1}{2})^n(g\pi \pm A)$	$(\frac{1}{2})^n(g\pi \pm B)$	$(\frac{1}{2})^n(g\pi \pm C)$.

The *upper* signs belong to *even* values of n and the *lower* to *odd* values of n, while g, the coefficient of π, is a number which it is impracticable if not impossible to express, but which is very easily derived from the corresponding coefficient in the preceding term and therefore easily tabulated.

Letting g stand for the coefficient of π in the n^{th} triangle, g' for the corresponding cheffìcient in the next preceding triangle, g'' in the next preceding &c., then

$$g = 2g' \pm 1, \quad g' = 2g'' \mp 1 \text{ \&c.,}$$

where the *upper* sign is to be used when g belongs to the *odd* triangle and the *lower* for the *even* triangle.

The following table gives the value of g for all values of n from 0 to 50.

n	g	n	g	n	g
0	0	17	43 691	34	6 399 391 061
1	1	18	87 381	35	12 798 782 123
2	1	19	175 763	36	25 597 564 245
3	3	20	351 525	37	51 195 128 491
4	5	21	703 051	38	102 390 256 981
5	11	22	1 406 101	39	204 780 513 963
6	21	23	2 812 203	40	409 561 027 925
7	43	24	5 624 405	41	819 122 052 851
8	85	25	11 248 811	42	1 638 244 111 701
9	171	26	22 497 621	43	3 276 488 223 403
10	341	27	44 995 243	44	6 552 976 446 805
11	683	28	99 990 485	45	13 105 952 893 611
12	1 365	29	199 980 971	46	26 211 905 787 221
13	2 731	30	399 961 941	47	52 423 811 574 443
14	5 461	31	799 923 883	48	104 847 623 148 885
15	10 923	32	1 599 847 765	49	209 695 246 297 771
16	21 845	33	3 199 695 531	50	419 390 492 595 541

Referring now to (1) we get by adding

$$s_1 = 2R\ (\cos\tfrac{1}{2}A + \cos\tfrac{1}{2}B + \cos\tfrac{1}{2}C). \tag{4}$$

Hencs $s_2 = 2R_1[\cos(\tfrac{1}{2})^2(\pi - A) + \cos(\tfrac{1}{2})^2(\pi - B) + \cos(\tfrac{1}{2})^2(\pi - C)]$

. . . .

$$s_{n-1} = 2R_{n-2}[\cos(\tfrac{1}{2})^{n-1}(g''\pi \pm A) + \cos(\tfrac{1}{2})^{n-1}(g''\pi \pm B) + \cos(\tfrac{1}{2})^{n-1}(g''\pi \pm C)] \tag{5'}$$

$$s_n = 2R_{n-1}[\cos(\tfrac{1}{2})^{n}(g'\pi \mp A) + \cos(\tfrac{1}{2})^{n}(g'\pi \mp B) + \cos(\tfrac{1}{2})^{n}(g'\pi \mp C)]. \tag{5}$$

It is well known that $R_1 = 2R$

whence $R_2 = 2R_1 = 2^2R$

$R_3 = 2R_2 = 2^3R$

. . . .

$$R_{n-1} = 2R_{n-2} = 2^{n-1}R \tag{6'}$$

$$R_n = 2R_{n-1} = 2^nR. \tag{6}$$

Substituting in (3) the values of s_{n-1} and $2R_{n-1}$ found in (5′) and (6) we have

$$K_n = 2^{2n-1}R^2[\cos(\tfrac{1}{2})^{n-1}(g''\pi \pm A) + \cos(\tfrac{1}{2})^{n-1}(g''\pi \pm B) + \cos(\tfrac{1}{2})^{n-1}(g''\pi \pm C)],$$

the argument for g'' being $n-2$.

Example. Suppose it is required to find the area of the 10th derived triangle, the original triangle being equilateral and its sides unity.

We have $R^2 = a^2 \div 4\sin^2 A = \frac{1}{3}$; $g'' = 85$ and the upper sign is to be used because n is *even*; hence

$$K_{10} = 2^{19}.\cos(\tfrac{1}{2})^9(85\pi + \tfrac{1}{3}\pi) = 2^{19}.\cos\tfrac{\pi}{6} = 2^{18}\sqrt{3}.$$

Cor. If we refer to the angles of the nth triangle we see that as g is increased the angles differ less and less from one another and hence if g is indefinitely increased the inequality of the angles is indefinitely diminished or the limiting triangle is equilateral.

SOLUTIONS OF PROBLEMS IN NUMBER SIX, VOL. IV.

Solutions of problems in No. 6, Vol. IV, have been rec'd as follows:

From Marcus Baker, 182 and 183; Prof. W. P. Casey, 182 and 186; Prof. A. B. Evans, 183; Geo. Eastwood, 183 and 185; Prof. J. M. Greenwood, 182, 183, 184 and 185; H. Heaton, 182, 183, 184, 185 and 186; W. E. Heal, 182 and 185; Prof. E. W. Hyde, 182, 183 and 186; Prof. W. W. Johnson, 186; Prof. J. H. Kershner, 182, 183 and 185; Christine Ladd, 186; Prof. D. J Mc. Adam, 182 and 185; Dr. A. B. Nelson, 183 and 185; Prof. A. T. Smith, 182; S. W. Salmon, 182, 183, 185, 186; Prof. Scheffer, 182, 183, 184, 185, 186; E. B. Seitz, 182—185, (for 186 see Chauv. Geom.)